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DWELL TIME FOR AN ASYMMETRIC ONE-DIMENSIONAL BARRIER

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We calculate the dwell times for incident particles comming from both the right and from the left of an asymmetric one–dimensional barrier. We prove that these times have a common contribution proportional to the density of states and an asymmetric contribution that depends on the reflection amplitudes from the right and from the left, which cancels in the symmetric case.

THERE HAS BEEN a great deal of interest in the study of the influence of the asymmetry of a potential barrier on the electronic transport properties and, in particular, on the resonant tunneling through a potential barrier [1–4]. In this case the mean dwell time τ_D, which is defined as the average time that the particle spends within the barrier, independently of whether it is ultimately transmitted or reflected, is an important time scale.

The dwell time was first introduced by Büttiker [5] as the ratio of the accumulated number of particles in the barrier to the incident flux:

$$\tau^{(D)} = \frac{1}{2k} \int_0^L |\psi(x)|^2 \, dx, \tag{1}$$

where the integral extends over the barrier, and $2k$ is the incident flux (we will take $e = c = \hbar = 1$, and $m_0 = 1/2$ for the electron mass). $\psi(x)$ is the steady-state scattering solution of the time-independent Schrödinger equation, whose energy dependence is not written explicitly.

As shown in papers [6, 7] the Büttiker's expression for a dwell time $\tau^{(D)}$ is correct in all case and does not depend on the approaches, which is usually the case in the theory of tunneling problems (see i.g. [8,9]. On the other hand, with the expression of the dwell time defined as a weighted average between the transmission and reflection times

$$\tau_D = T\tau_T + R\tau_R \tag{2}$$

we have a many problems (see, e.g. [7]). For example, the phase time for transmission and reflection of Bohm and Wigner do not satisfy Eq. (2).

Our previous results [9], where we determine the dwell time for a symmetric barrier in terms of the Green function, and prove that it is proportional to the density of states, also are in contradiction with the dwell time defined by Eq. (2).

The purpose of this work is to calculate directly the dwell time from Eq. (1) for the case of a general one-dimensional asymmetric barrier in terms of the density of states of system.

Let us consider a particle moving along the x–direction in the presence of an arbitrary potential barrier $V(x)$ in the interval $(0, L)$. The potential is zero outside the barrier. Our aim is to calculate the dwell time, given by Eq. (1), for particles coming both from the left and from the right. We evaluate Eq. (1) in three steps. First, we incorporate the fact that the wavefunction appearing in this equation is a solution of the Schrödinger equation. Second, we rewrite the wavefunctions in terms of Green functions. And finally, we express the Green functions in terms of the density of states and the reflection coefficients.

First of all, we can trivially rewrite the wavefunction $\psi(x)$, which is a solution of the Schrödinger equation, as:

$$\psi(x) = (V(x) - E)\frac{\partial}{\partial E}\psi(x) - \frac{\partial}{\partial E}(V(x) - E)\,\psi(x). \tag{3}$$

Then the modulus square $|\psi(x)|^2$ takes the following

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form:

$$|\psi(x)|^2 = \frac{1}{2}\frac{\partial}{\partial x}\left\{\left(\psi'(x)\frac{\partial}{\partial E}\psi^*(x) + \psi^{*\prime}(x)\frac{\partial}{\partial E}\psi(x)\right) - \left(\psi(x)\frac{\partial}{\partial E}\psi^{*\prime}(x) + \psi^*(x)\frac{\partial}{\partial E}\psi'(x)\right)\right\}. \quad (4)$$

Integrating this expression and dividing by the incident flux, $2k = 2\sqrt{E}$, we obtain:

$$\frac{1}{2k}\int_0^L |\psi(x)|^2 dx = -\frac{1}{4k}\left[\psi^{*2}(x)\frac{\partial}{\partial E}\left(\frac{\psi'(x)\psi(x)}{|\psi(x)|^2}\right) + \psi^2(x)\frac{\partial}{\partial E}\left(\frac{\psi^{*\prime}(x)\psi^*(x)}{|\psi(x)|^2}\right)\right]_0^L. \quad (5)$$

Here the prime signifies the derivative with respect to the x coordinate. This expression is formally the same for particles incident from the left or from the right, but we have to remember that the corresponding wavefunctions will not be the same. García-Calderón and Rubio [10] arrived at the same result by a completely different method.

Our second step is to rewrite Eq. (5) in terms of the retarded Green function $G(x, x')$ (GF) of the system, that we will use throughout the communication. The wavefunction $\psi(x)$ at energy E is related to the GF through the expression:

$$G(x, x') = \begin{cases} i\pi\nu(E)\psi(x)\psi^*(x') & \text{if } x > x' \\ i\pi\nu(E)\psi^*(x)\psi(x') & \text{if } x \le x' \end{cases}. \quad (6)$$

where $\nu(E)$ is the density of states per unit energy and per unit length. At coinciding coordinates, this expression reduces to the well-known result $G(x, x) = i\nu(E)|\psi(x)|^2$. From Eq. (6) we can obtain the left-hand side and right-hand side derivatives of the GF with respect to coordinates, which have to be distinguished due to discontinuity:

$$\dot{G}(x \mp 0, x) = \pm\frac{1}{2} + \frac{1}{2}G'(x, x). \quad (7)$$

Here the dot signifies the derivative with respect to the first argument and the prime the derivative with respect to the two arguments simultaneously. Taking into account these expressions we can write the first factor in the RHS of Eq. (5) containing $\partial/\partial E$ as:

$$\frac{\partial}{\partial E}\left(\frac{\psi'(x)\psi(x)}{|\psi(x)|^2}\right) = \frac{\partial}{\partial E}\left(\frac{\dot{G}(x+0, x)}{G(x, x)}\frac{\psi(x)}{\psi(x)^*}\right) = \frac{\partial}{\partial E}\left(\frac{-1 + G'(x, x)}{2G(x, x)}e^{2i\theta(x,E)}\right) \quad (8)$$

where $\theta(x, E)$ is a phase function and is defined by

$$\theta(x, E) = \exp\left\{-\int_{\min(x,x')}^{\max(x,x')}\frac{dx_1}{2G(x_1, x_1)}\right\}. \quad (9)$$

A similar expression is valid for the other factor in Eq. (5) containing $\partial/\partial E$. Thus, the dwell time can be written in terms of GF as:

$$\tau^{(D)} = \left[|G(0, x)|^2\left\{\frac{i}{G(x, x)}\frac{\partial}{\partial E}\theta(x, E) - \frac{\partial}{\partial E}\left(\frac{G'(x, x)}{G(x, x)}\right)\right\}\right]_0^L. \quad (10)$$

As for the wavefunction, the GF $G(x, x')$ depends on whether the particle arrives at the barrier from the left or from the right.

The GF for an arbitrary stepwise barrier was obtained by Aronov *et al.* [11] and their results can be generalized to an arbitrary barrier by considering an infinite number of steps. This technique was already applied by us [13] to obtain the tunneling time of an arbitrary barrier. After some cumbersome algebra we arrive at:

$$\tau_-^{(D)} = \frac{1}{2k}\mathrm{Im}\left\{\frac{\partial}{\partial k}\ln t + \frac{1}{2k}(R_- + R_+)\right\} + \frac{1}{4k}\mathrm{Im}\left\{R\frac{\partial}{\partial k}\ln\frac{R_-}{R_+} + \frac{1}{k}(R_- - R_+)\right\}. \quad (11)$$

The subscript - indicates that the particle is comming from the left. R_- and R_+ are the reflection amplitudes from the left and from the right, respectively, R is the modulus of these amplitudes $R = |R_-| = |R_+|$, and t is the transmission amplitude, which is independent of the incident direction as can be deduced from the time-reversal and current conservation requirements [12]. A similar expression to (11) holds when the particle is coming from the right, interchanging R_- and R_+. We will refer to this case with the subindex +. The reflection and transmission amplitudes can be calculated with any of the available techniques, such us the transfer matrix technique or the characteristic determinant method.

We showed [13] that the first term on the RHS of the last equation is proportional to the density of states. Then, we finally arrive at the following expression for the dwell time:

$$\tau_\pm^{(D)} = \pi L\nu(E) + \frac{1}{4k}\mathrm{Im}\left\{R\frac{\partial}{\partial k}\ln\frac{R_\mp}{R_\pm} + \frac{1}{k}(R_\mp - R_\pm)\right\}. \quad (12)$$

For a symmetric potential we have $R_- = R_+$ and we obtain $\tau_-^{(D)} = \tau_+^{(D)} = \nu(E)$, in agreement with our previous result [9,14].

For an asymmetric barrier, it is easy to check that the contribution from the asymmetry is the opposite for particles comming from the left and from the right. Then we find that:

$$\nu(E) = \frac{1}{2\pi L}\left(\tau_{-}^{(D)} + \tau_{+}^{(D)}\right). \tag{13}$$

This result was obtained in a much wider context by Iannaccone [15].

We have obtained an exact and general expression of the dwell time for an asymmetric barrier in terms of the density of states and the reflection amplitudes from the left and from the right.

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